D2D Assisted Coded Caching Design for Multi-Access Networks

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***Abstract*—This letter explores a device-to-device (D2D) assisted setting in the multi-access networks, where the users communicate with each other and each one can access multiple neighboring cache nodes in a cyclic wrap-around fashion. For such networks, we introduce two new combinatorial arrays namely the caching array and the delivery array to characterize the coded caching process. Based on this, a novel construction of caching array and delivery array is derived through a smart transform over the existing placement delivery array (PDA). The resulting coded caching scheme can achieve a small transmission rate under the case of uncoded placement. It is shown that the proposed scheme has advantages in both the transmission rate and the subpacketization level over the benchmark scheme.**

***Index Terms*—Coded caching, device-to-device communication, multi-access networks, transmission rate.**

I. Introduction

The growing demands on video streaming services lead to a severe network congestion during the peak traffic times. Coded caching has been introduced as an effective technique to alleviate the burden of high network load by leveraging the potential coded multi-cast opportunities across the network. The original coded caching network [1] consists of a central server which has access to a library of N equal size files. It is connected by a set of K users through a noiseless shared link, and each user is equipped with a dedicate cache with a size of M files. This technique involves two phases. In the placement phase, the server populates the appropriate contents to each user's cache without any prior knowledge of the later demands. Afterwards, in the delivery phase, the server will be informed with the users' demands, and then broadcast some potentially coded messages to the users so that their demands can be satisfied.

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It is demonstrated that the transmission rate of the scheme in [1], which we refer to as the MN scheme, is optimal under the constraints of uncoded placement and $K \leq N$ [2]. Following the seminal work of [1], many later researches have dedicated to the improvements in terms of the achievable rates and lower bounds, e.g., if the library file is requested by multiple users, an improved transmission rate of the MN scheme was derived in [3]. Meanwhile, the work of [4] proposed a coded caching scheme that achieves the best memory-rate tradeoff under the constraint of coded placement. Moreover, several theoretical lower bounds on the transmission rate was characterized in [5], [6]. However, the above mentioned works require each library file to be partitioned into F packets (i.e., subpacketization level), where F increases exponentially with the number of users. This is impractical for large networks. The exponentially growing subpacketization issue has been addressed in [7], [8], [9]. However, a reduction of subpacketization level is often realized at the cost of the transmission rate.

Recently, inspired by the circular Wyner model for interference networks [10], the multi-access coded caching network was investigated in [11], where each user is connected by multiple neighbouring cache nodes in a cyclic wrap-around fashion. Based on this, various types of multi-access coded caching schemes were proposed through combinatorial methodologies [12], [13], [14]. In particular, by means of index coding technique, a multi-access coded caching scheme that yields a small transmission rate was proposed in [12]. In [13], a transformation approach was introduced in realizing a multi-access coded caching scheme with flexible system parameters. Coded caching has also been studied in device-to-device (D2D) networks that enable the users to communicate with each other such that the delivery load can be further reduced. In [15], an exact characterization of the optimal rate-memory trade off under the case of uncoded placement was provided. The work of [16] introduced a combinatorial array called D2D placement delivery array (DPDA) to characterize its coded caching process. The work of [17] studied a D2D network where only a part of users participate in delivering the missing packets to network users. In recent years, D2D assisted coded caching has also been extended to some more practical scenarios, including distinct cache sizes [18], private caches [19], wireless multi-hop networks [20], and mobility-aware networks [21]. However, D2D networks in which each user accesses multiple cache nodes have not been discussed in the literature.

This letter formulates a D2D assisted multi-access coded caching network. Two new combinatorial arrays called the caching array and the delivery array are introduced to characterize its coded caching process. In order to minimize the

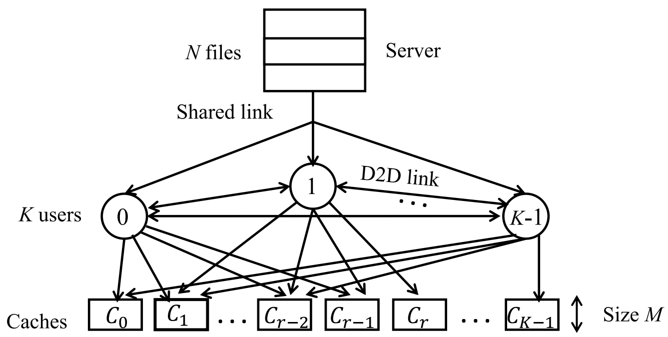

Fig. 1. A (K, r, M, N) D2D assisted multi-access coded caching network.

transmission rate in the delivery phase, a novel transformation approach is utilized in the design of caching array and delivery array. Consequently, the resulting scheme can achieve a good rate-memory tradeoff under the case of uncoded placement. It is shown that the proposed scheme can yield a smaller subpacketization level and also a smaller transmission rate over the existing D2D baseline scheme of [16].

Notations: Let calligraphic symbols and bolded lower-case letters denote sets and vectors, respectively. Let $|\cdot|$ denote the cardinality of a set. Set of consecutive integers is represented by $[a:b] = \{a, a+1, \ldots, b\}$. Let $\binom{[a:b]}{t}$ denote the collection of all size-t subsets of $[a:b]$. Let $<a>_b$ denote the least non-negative residue of a modulo b. Symbol $<[a:b]>_q$ represents each integer of $[a:b]$ modulo q, i.e., $<[a:b]>_q = \{<a>_q, <a+1>_q, \ldots, <b>_q\}$. Given an array $\mathbf{D}$, let $\mathbf{D}(f, k)$ denote its entry of row f and column k. Finally, let $[\mathbf{D}_0; \mathbf{D}_1; \ldots; \mathbf{D}_{n-1}]$ represent an array obtained by arranging arrays $\mathbf{D}_0, \mathbf{D}_1, \ldots, \mathbf{D}_{n-1}$ from top to bottom.

II. Network Model and Background

This section presents the D2D assisted multi-access coded caching network model, and reviews the DPDA of [16].

A. Network Model

We consider a (K; r; M; N) D2D assisted multi-access coded caching network, as shown in Fig. 1. A central server with N equal size files, denoted by $\mathcal{W} = \{W_0, W_1, \ldots, W_{N-1}\}$, is connected to K users through a noiseless shared link, where the users are not equipped with caches. The users can be connected with each other over noiseless D2D communication links. There are K cache-nodes $C_0, C_1, \ldots, C_{K-1}$, each of which has a cache size of M files, where $M \le \frac{N}{r}$. Each user can retrieve r consecutive cache nodes in a cyclic wrap-around fashion, i.e., user k is connected to the cache nodes $C_{<k>_K}, C_{<k+1>_K}, \ldots, C_{<k+r-1>_K}$, where $k \in [0:K-1]$. The network operates in two phases as follows.

- *Placement Phase:* Each library file is partitioned into F equal packets, i.e., $W_n = \{W_{n,f} | f \in [0:F-1]\}$, $n \in [0:N-1]$. Based on a specific cache placement strategy, each cache node C_k is populated with some packets without any prior knowledge of the user demands, where $k \in [0:K-1]$. The size of C_k cannot be greater than its cache size M.
- *Delivery Phase:* Each user requests an arbitrary file from $\mathcal{W}$. The request vector is denoted by $\mathbf{d} = (d_0, d_1, \ldots, d_{K-1})$, i.e., user k requests file W_{d_k}, where $d_k \in [0:N-1]$. Once the users' requests are revealed, the signals of size at most RF packets will be delivered through the D2D communication links so that each user's demand can be satisfied with the help of its available cached contents, where R is called the transmission rate. It requires the sum of the users' accessing cache content sizes being lager than N files, i.e., $KrM \ge N$. Our main objective is to design a coded scheme that supports D2D assisted multi-access networks with R being as small as possible.

B. D2D Placement Delivery Array

This subsection briefly reviews the definition of DPDA and its relationship to a D2D coded caching scheme.

Definition 1 [16]: Given positive integers K, F, Z and S, an $F \times K$ array $\mathbf{D} = (\mathbf{D}(f, k))$, where $f \in [0:F-1]$, $k \in [0:K-1]$, and $\mathbf{D}(f, k) \in [0:S-1] \cup \{*\}$, is called a ($K$, F, Z, S) DPDA if the following conditions are satisfied:

C1. Each column of $[0:K-1]$ has exactly Z "$*$"s;

C2. Each integer of $[0:S-1]$ appears at least once in the array;

C3. For any two distinct entries $\mathbf{D}(f_0, k_0)$ and $\mathbf{D}(f_1, k_1)$, $\mathbf{D}(f_0, k_0) = \mathbf{D}(f_1, k_1) = s$ is an integer only if (a) $f_0 \ne f_1, k_0 \ne k_1$, i.e., they lie in distinct rows and distinct columns, and (b) $\mathbf{D}(f_1, k_0) = \mathbf{D}(f_0, k_1) = *$;

C4. There exists a mapping ϕ defined over $[0:S-1]$ to $[0:K-1]$ such that $\mathbf{D}(f, k) = s$ for any $s \in [0:S-1]$, then $\mathbf{D}(f, \phi(s)) = *$.

In particular, if $\mathbf{D}$ satisfies Conditions C1, C2 and C3, it is called a placement delivery array (PDA) [7]. Note that if each integer of a PDA appears g times, it is referred to as a g-regular PDA, denoted by g-(K, F, Z, S) PDA, e.g., the following array is a 2-(4, 4, 2, 4) DPDA, since it satisfies all the conditions of *Definition 1*.

$$\mathbf{D} = \begin{pmatrix} 0 & * & * & 2 \\ * & 0 & * & 3 \\ 1 & * & 2 & * \\ * & 1 & 3 & * \end{pmatrix} \tag{1}$$

D2D communication networks can be realized by the above DPDA. Given a (K, F, Z, S) DPDA $\mathbf{D}$, its rows and columns represent the packet indices and cache node indices, respectively. If $\mathbf{D}(f, k) = *$, cache node C_k will store the fth packet of all the files. Condition C1 of *Definition 1* implies that each cache node has a cache size of $M = \frac{NZ}{F}$ files. $\mathbf{D}(f, k) = s$ implies that the fth packet of all the files is not placed at cache node C_k. The XOR of the requested packets $\bigoplus_{\mathbf{D}(f,k)=s, f\in[0:F-1], k\in[0:K-1]} W_{d_k, f}$ will be broadcast over the D2D communication links. Note that user k is only connected by cache node C_k in the original D2D communication networks. Conditions C3 and C4 of *Definition 1* guarantee that user k can get its desired packet, since it has cached all the other packets in the coded message except the desired one. Finally, Condition C2 of *Definition 1* implies that the number of transmitted packets is exactly S and the transmission rate is $R = \frac{S}{F}$. Therefore, the following result can be obtained.

Lemma 1 [16]: Given a (K, F, Z, S) DPDA $\mathbf{D}$, there always exists a (K, M, N) D2D coded caching scheme with the cache memory size of $M = \frac{NZ}{F}$, the transmission rate of $R = \frac{S}{F}$ and the subpacketization level of F.

The following example demonstrates this property.

Example 1: Given the DPDA $\mathbf{D}$ of (1), and based on *Lemma 1*, a (4, 2, 4) D2D coded caching scheme can be obtained as

- *Placement Phase:* Each file W_n is partitioned into four packets, i.e., $W_n = \{W_{n,0}, W_{n,1}, W_{n,2}, W_{n,3}\}$, where $n \in [0:3]$. The contents placed at each cache node are $\mathcal{Z}_{C_0} = \{W_{n,1}, W_{n,3} \mid n \in [0:3]\}$, $\mathcal{Z}_{C_1} = \{W_{n,0}, W_{n,2} \mid n \in [0:3]\}$ $\mathcal{Z}_{C_2} = \{W_{n,0}, W_{n,1} \mid n \in [0:3]\}$ and $\mathcal{Z}_{C_3} = \{W_{n,2}, W_{n,3} \mid n \in [0:3]\}$.
- *Delivery Phase:* Let us assume that the request vector is $\mathbf{d} = (0, 1, 2, 3)$. The signals sent by four users are: User 0 : $W_{2,3} \oplus W_{3,1}$; User 1: $W_{2,2} \oplus W_{3,0}$; User 2: $W_{0,0} \oplus W_{1,1}$; User 3: $W_{0,2} \oplus W_{1,3}$. Each user can then reconstruct its required file, e.g., user 0 requires W_0 and it can access $W_{0,1}$ and $W_{0,3}$. It can obtain $W_{0,0}$ with the received coded packet $W_{0,0} \oplus W_{1,1}$, where $W_{1,1}$ is available. Moreover, it can also obtain $W_{0,2}$ with the received coded packet $W_{0,2} \oplus W_{1,3}$, where $W_{1,3}$ can be retrieved. Hence, the transmission rate is $R = \frac{4}{4} = 1$.

III. The Proposed D2D Assisted Multi-Access Coded Caching Scheme

This section proposes a new coded caching scheme for D2D assisted multi-access networks. Similar to DPDA, the proposed scheme can be realized by a caching array $\mathbf{C}$ and a delivery array $\mathbf{D}$. Caching array $\mathbf{C}$ consists of "$*$"s and nulls, whose rows and columns denote the packet indices and cache node indices, respectively. If $\mathbf{C}(f, k) = *$, the fth packet of each file is placed at cache node C_k. $\mathbf{D}(f, k) = *$ implies that the fth packet of each file can be retrieved by user k. In order to guarantee that each user's demand can be satisfied, the delivery array $\mathbf{D}$ should be well designed such that it is a DPDA. Next, we will present a new design of caching array $\mathbf{C}$ and delivery array $\mathbf{D}$ based on an existing regular PDA $\mathbf{P}$ that further satisfies the following conditions.

C5. The number of "$*$"s in each row is a constant;

C6. If $\mathbf{P}(f_0, k_0) = \mathbf{P}(f_1, k_1) = s$ and $k_0 > k_1$, then $|\{k \mid \mathbf{P}(f_0, k) = *, k < k_0\}| \geq |\{k \mid \mathbf{P}(f_1, k) = *, k < k_1\}|$.

Construction 1: Given a positive integer r $(r > 1)$ and a g-(K', F', Z', S') PDA $\mathbf{P}$ satisfying Condition C6 with each row having t "$*$"s, an $F'(g-1)[(r-1)K'+t] \times [(r-1)K'+t]$ caching array $\mathbf{C} = [\mathbf{C}_0; \mathbf{C}_1; \ldots; \mathbf{C}_{(r-1)K'+t-1}]$ can be constructed with the entries of $\mathbf{C}_b$, where $b \in [0:(r-1)K'+t-1]$, defined as

$$\mathbf{C}_b(f,k) = \begin{cases} *, & \text{if } \mathbf{P}(<f>_{F'}, j) = *, k = <jr+b+r \\ & -1-\xi>_{(r-1)K'+t}, \text{ and there are } \xi \\ & \text{columns } j' \text{ such that } \mathbf{P}\big(<f>_{F'}, j'\big) \neq \\ & * \text{ for } j' < j; \\ \text{null}, & \text{otherwise.} \end{cases} \tag{2}$$

Note that $f \in [0:F'(g-1)-1]$, $k \in [0:(r-1)K'+t-1]$ and $j \in [0:K'-1]$. Let $\mathcal{F}_s = \{f \mid \mathbf{P}(<f>_{F'}, j) = s, f \in [0:F'(g-1)-1], j \in [0:K'-1]\}$ and its elements are arranged in an ascending order. The elements of $\mathcal{F}_s$ are further partitioned into g groups from left to right with the same size of $g-1$, i.e., $\mathcal{F}_s = \{\mathcal{F}_s^{(0)}, \mathcal{F}_s^{(1)}, \ldots, \mathcal{F}_s^{(g-1)}\}$. Let $\mathcal{F}'_s = \{f \mid \mathbf{P}(f,j) = s, f \in [0:F'-1], j \in [0 \quad K'-1]\}$. Based on the caching array $\mathbf{C}$, an $F'(g-1)[(r-1)K'+t] \times [(r-1)K'+t]$ delivery array $\mathbf{D} = [\mathbf{D}_0; \mathbf{D}_1; \ldots; \mathbf{D}_{(r-1)K'+t-1}]$ can be obtained with the entries of $\mathbf{D}_b$ defined as

$$\mathbf{D}_b(f,k) = \begin{cases} \mathbf{y}, \text{ if } \mathbf{P}(<f>_{F'}, j) = s, k = <j(r-1)+\beta \\ \quad +\eta>_{(r-1)K'+t}, \beta \in [b : b+r-2], f \in \mathcal{F}_s^{(\alpha)} \\ \quad \text{for } \alpha \in [0:g-1] \text{ and there are } \eta \text{ columns} \\ \quad j' \text{ such that } \mathbf{P}\big(f, j'\big) = * \text{ for } j' < j; \\ *, \text{ otherwise,} \end{cases} \tag{3}$$

where $\mathbf{y} = (s + S'(\beta - b) + b(r-1)S', \mathcal{F}'_s \backslash <\mathcal{F}_s^{(\alpha)}>_{F'})$.

The following example is further developed to illustrate the above construction.

Example 2: Given $r = 3$, let us consider the following 3-(4, 6, 3, 4) PDA $\mathbf{P}$ that satisfies Conditions C5 and C6. Based on (2), we have $\mathbf{C}_0(0,2) = *$. This is because $\mathbf{P}(<0>_6, 0) = *$ and $<jr+b+r-1-\xi>_{(r-1)K'+t} = <2>_{10} = 2$ due to $\xi = 0$. Similarly, with $\mathbf{P}(<1>_6, 2) = *$ and $<jr+b+r-1-\xi>_{(r-1)K'+t} = <7>_{10} = 7$ due to $\xi = 1$, we can conclude that $\mathbf{C}_0(1,7) = *$. Similarly, we can obtain $\mathbf{C}_0(f,k)$ for any $f \in [0:11]$ and $k \in [0:9]$. As a result, the array $\mathbf{C}_0$ can be obtained as follows. With the same construction, the caching array $\mathbf{C} = [\mathbf{C}_0; \mathbf{C}_1; \ldots; \mathbf{C}_9]$ can also be derived. Based on the caching array $\mathbf{C}$, it can be seen that any r neighboring cache nodes do not cache any common packets.

	0	1	2	3
0	*	*	0	1
1	*	0	*	2
2	*	1	2	*
3	0	*	*	3
4	1	*	3	*
5	2	3	*	*

3-(4, 6, 3, 4) MN PDA $\mathbf{P}$

	0	1	2	3	4	5	6	7	8	9
0			*			*				
1			*					*		
2			*							*
3					*			*		
4					*					*
5							*			*
6			*			*				
7			*					*		
8			*							*
9					*			*		
10					*					*
11							*			*

$\mathbf{C}_0$: The first part of caching array $\mathbf{C}$ obtained from $\mathbf{P}$

	0	1	2	3	4	5	6	7	8	9
0	*	*	*	*	*	*	(0,3)	(4,3)	(1,4)	(5,4)
1	*	*	*	(0,3)	(4,3)	*	*	*	(2,5)	(6,5)
2	*	*	*	(1,4)	(5,4)	(2,5)	(6,5)	*	*	*
3	(0,1)	(4,1)	*	*	*	*	*	*	(3,5)	(7,5)
4	(1,2)	(5,2)	*	*	*	(3,5)	(7,5)	*	*	*
5	(2,2)	(6,2)	(3,4)	(7,4)	*	*	*	*	*	*
6	*	*	*	*	*	*	(0,1)	(4,1)	(1,2)	(5,2)
7	*	*	*	(0,0)	(4,0)	*	*	*	(2,2)	(6,2)
8	*	*	*	(1,0)	(5,0)	(2,1)	(6,1)	*	*	*
9	(0,0)	(4,0)	*	*	*	*	*	*	(3,4)	(7,4)
10	(1,0)	(5,0)	*	*	*	(3,3)	(7,3)	*	*	*
11	(2,1)	(6,1)	(3,3)	(7,3)	*	*	*	*	*	*

$\mathbf{D}_0$: The first part of delivery array $\mathbf{D}$ obtained from $\mathbf{P}$

The delivery array $\mathbf{D} = [\mathbf{D}_0; \mathbf{D}_1; \ldots; \mathbf{D}_9]$ can be further constructed based on (3). Without loss of generality, let us consider the construction of $\mathbf{D}_0$. Since $\mathbf{P}(<0>_6, 2) = 0$, and for $\beta = 0$, $<j(r-1)+\beta+\eta>_{(r-1)K'+t} = <6>_{10} = 6$ due to $\eta = 2$, it can be seen that the first coordinate of entry

$\mathbf{D}_0(0,6)$ is $s+S'(\beta-b)+b(r-1)S'=0$. Note that $\mathcal{F}'_0=\{0,1,3\}$, $\mathcal{F}_0=\{\mathcal{F}_0^{(0)},\mathcal{F}_0^{(1)},\mathcal{F}_0^{(2)}\}=\{\{0,1\},\{3,6\},\{7,9\}\}$ and $f=0\in\mathcal{F}_0^{(0)}$. This implies that $<\mathcal{F}_0^{(0)}>_6=\{<0>_6,<1>_6\}=\{0,1\}$ and $\mathcal{F}'_0\setminus<\mathcal{F}_0^{(0)}>_6=\{3\}$, i.e., the second coordinate of entry $\mathbf{D}_0(0,6)$ is 3. Therefore, we have $\mathbf{D}_0(0,6)=(0,3)$. Similarly, we can obtain $\mathbf{D}_0(f,k)$ for any $f\in[0:11]$ and $k\in[0:9]$. As a result, the above array $\mathbf{D}_0$ can be obtained. It can be seen that $\mathbf{D}$ satisfies Condition C4 of *Definition 1* and each user can access r neighbouring cache nodes in a cyclic wrap-around manner.

Based on *Construction 1*, a new D2D assisted multi-access coded caching scheme that is characterized by the following theorem can be obtained.

Theorem 1: Given a positive integer r ($r>1$) and g-(K',F',Z',S') PDA satisfying Condition C6 with each row having t "$*$"s, there always exists a (K, r, M, N) D2D assisted multi-access coded caching scheme that supports $K=(r-1)K'+t$ users. Its cache node has a memory size of $M=\frac{Nt}{(r-1)K'+t}$. Its transmission rate is $R=\frac{S'g(r-1)}{F'(g-1)}$ and subpacketization level is $F=F'(g-1)[(r-1)K'+t]$.

Proof: Based on (2), it can be seen that each row of $\mathbf{C}_b$ has t "$*$"s. This implies that the total number of "$*$"s in $\mathbf{C}_b$ is $tF'(g-1)$. Since $k=<jr+b+r-1-\xi>_{(r-1)K'+t}$ for the fixed f, it can be seen that the "$*$"s located in row f of $\mathbf{C}_{b+1}$ are obtained by cyclically shifting the "$*$"s in row f of $\mathbf{C}_b$ to the right by one position. This implies that each column of caching array $\mathbf{C}$ has $F't(g-1)$ "$*$"s, i.e., the cache memory size of each cache node is $M=\frac{NZ}{F}=\frac{Nt}{[(r-1)K'+t]}$, which satisfies $KrM\geq N$. In the following, we will show that the delivery array $\mathbf{D}$ is a DPDA.

With a similar analysis above, based on (3), it can be seen that each column of $\mathbf{D}$ has $F'rt(g-1)$ "$*$"s, i.e., Condition C1 of *Definition 1* holds. In order to show that $\mathbf{D}$ satisfies Condition C3 of *Definition 1*, it needs to prove that $\mathbf{D}_b$ satisfies Condition C3 of *Definition 1*. This is because each entry of $\mathbf{D}_{b+1}$ is different with the entries of $\mathbf{D}_b$ based on (3). Let us assume that $\mathbf{D}_b(f_0,k_0)=\mathbf{D}_b(f_1,k_1)=(s+S'(\beta-b)+b(r-1)S',\mathcal{F}'_s\setminus<\mathcal{F}_s^{(\alpha)}>_{F'})$. Based on (3), it can be seen that $f_0,f_1\in\mathcal{F}_s^{(\alpha)}$ and $f_0\neq f_1$, where $\alpha\in[0:g-1]$. Note that $k_0=<j_0(r-1)+\beta+\eta_0>_{(r-1)K'+t}$ and $k_1=<j_1(r-1)+\beta+\eta_1>_{(r-1)K'+t}$, where j_0 and j_1 satisfy $\mathbf{P}(<f_0>_{F'},j_0)=\mathbf{P}(<f_1>_{F'},j_1)=s$. Without loss of generality, let us assume that $j_0>j_1$. Based on Condition C6, since $\eta_0\geq\eta_1$, we have $k_0\neq k_1$. Therefore, Condition C3 (a) of *Definition 1* holds. Furthermore, let us assume that $\mathbf{D}_b(f_0,k_1)\neq *$. Based on (3), we have

$$\begin{aligned}k_1&=<j_1(r-1)+\beta+\eta_1>_{(r-1)K'+t}\\&=<j'(r-1)+\beta'+\eta'>_{(r-1)K'+t},\end{aligned}\tag{4}$$

where j' satisfies $\mathbf{P}(<f_0>_{F'},j')=s'$. If $j'<j_1$, we have $\eta'\leq\eta_1$. This contradicts the above equation (4), since $\beta'-\beta\leq r-2$ due to $\beta'\in[b:b+r-2]$. If $j'>j_1$, we have $\eta'\geq\eta_1$. This also contradicts equation (4), since $\beta-\beta'\leq r-2$ due to $\beta'\in[b:b+r-2]$. Therefore, we have $j'=j_1$, which implies that $\mathbf{P}(<f_0>_{F'},j_1)=s'$. This is impossible, since $\mathbf{P}$ is a PDA and $\mathbf{P}(<f_0>_{F'},j_1)=*$. Hence, we have $\mathbf{D}_b(f_0,k_1)=*$. Similarly, we can also show that $\mathbf{D}_b(f_1,k_0)=*$. Therefore, Condition C3 (b) of *Definition 1* holds.

Suppose that $\mathbf{D}_b(f_h,k_h)=(s+S'(\beta-b)+b(r-1)S',\mathcal{F}'_s\setminus<\mathcal{F}_s^{(\alpha)}>_{F'})$, where $h\in[0:g-1]$ and $k_h=<j_h(r-1)+\beta+\eta_h>_{(r-1)K'+t}$. Based on (3), it can be seen that $\mathbf{D}_h(f_h,<jr+b+r-1-\xi>_{(r-1)K'+t})=*$, where j satisfies $\mathbf{P}(\mathcal{F}'_s\setminus<\mathcal{F}_s^{(\alpha)}>_{F'},j)=s$. This is because for $\beta\in[b:b+r-2]$, we have $<j(r-1)+\beta+\eta>_{(r-1)K'+t}\neq<jr+b+r-1-\xi>_{(r-1)K'+t}$ due to $j=\eta+\xi$. Therefore, Condition C4 of *Definition 1* holds. Further based on (3), it can be seen that the number of distinct entries in $\mathbf{D}_b$ is $S'(r-1)g$. This implies that the number of distinct entries of $\mathbf{D}$ is $S'(r-1)g[(r-1)K'+t]$. Hence, $\mathbf{D}$ is a $((r-1)K'+t,F'(g-1)[(r-1)K'+t],F'rt(g-1),S'(r-1)g[(r-1)K'+t])$ DPDA. It can realize a D2D assisted coded caching scheme with the transmission rate of $R=\frac{S'(r-1)g[(r-1)K'+t]}{F'(g-1)[(r-1)K'+t]}=\frac{S'g(r-1)}{F'(g-1)}$ and the subpacketization level of $F=F'(g-1)[(r-1)K'+t]$. ∎

Let us briefly review the MN PDA of [1]. Given a positive integer K and $t\in[1:K-1]$, a $(t+1)$-$(K,\binom{K}{t},\binom{K-1}{t-1},\binom{K}{t+1})$ MN PDA $\mathbf{P}$ with its entry $\mathbf{P}(\mathcal{A},k)$, where $\mathcal{A}\in\binom{[0:K-1]}{t}$ and $k\in[0:K-1]$, is defined as

$$\mathbf{P}(\mathcal{A},k)=\begin{cases}\varphi(\mathcal{A}\cup\{k\}), & \text{if } k\notin\mathcal{A};\\ *, & \text{otherwise,}\end{cases}\tag{5}$$

where φ is an order function over $\binom{[0:K-1]}{t+1}$ and each size-$t{+}1$ element of $\binom{[0:K-1]}{t+1}$ is arranged in the lexicographic order. The following lemma shows the property of the MN PDA.

Lemma 2: The MN PDA satisfies Conditions C5 and C6.

Proof: Based on (5), it can be seen that each row of the MN PDA has t "$*$"s. Suppose that $\mathbf{P}(\mathcal{A}_0,k_0)=\mathbf{P}(\mathcal{A}_1,k_1)=s$ and $k_0>k_1$, where $k_0\notin\mathcal{A}_0$ and $k_1\notin\mathcal{A}_1$. Let $\mathcal{B}_{\mathcal{A}_0}=\{k\mid\mathbf{P}(\mathcal{A}_0,k)=*,k<k_0\}$ and $\mathcal{B}_{\mathcal{A}_1}=\{k\mid\mathbf{P}(\mathcal{A}_1,k)=*,k<k_1\}$. Based on (5), it can be seen that $\mathcal{A}_0\cup\{k_0\}=\mathcal{A}_1\cup\{k_1\}$. This implies that $\mathcal{A}_0\setminus\{k_0,k_1\}=\mathcal{A}_1\setminus\{k_0,k_1\}$. Therefore, we have $|\mathcal{B}_{\mathcal{A}_0}|=|\mathcal{B}_{\mathcal{A}_1}|+1$ due to $\mathbf{P}(\mathcal{A}_0,k_1)=*$. ∎

In fact, most of the existing regular PDAs satisfy Conditions C5 and C6, such as the works of [8], [9]. Both *Theorem 1* and *Lemma 2* lead to the following conclusion.

Theorem 2: Given a positive integer r ($r>1$) and a (t_0+1)-$(K_0,\binom{K_0}{t_0},\binom{K_0-1}{t_0-1},\binom{K_0}{t_0+1})$ MN PDA $\mathbf{P}$, there always exists a (K, r, M, N) D2D assisted multi-access coded caching scheme with the user number of $K=(r-1)K_0+t_0$, the cache node memory size of $M=\frac{Nt_0}{(r-1)K_0+t_0}$, the transmission rate of $R=\frac{\binom{K_0}{t_0+1}(t_0+1)(r-1)}{\binom{K_0}{t_0}t_0}$ and the subpacketization level of $F=\binom{K_0}{t_0}t_0[(r-1)K_0+t_0]$.

IV. Performance Analysis

This section analyzes the proposed coded caching scheme in terms of the transmission rate and subpacketization level. It is first compared with the baseline scheme obtained from [16]. It was shown that in [16], any regular PDA can be transformed into a DPDA. Note that the delivery array designed for multi-access networks of [13] is a regular PDA. Therefore, a baseline D2D assisted multi-access coded caching scheme can be obtained in the following lemma.

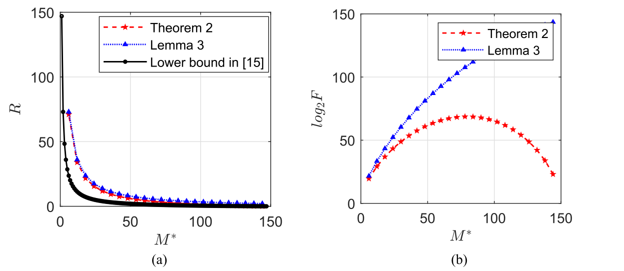

Fig. 2. Comparison between the proposed schemes of *Theorem 2*, *Lemma 3* and [15], where $r = 3$ and $K = N = 148$. (a) Transmission rate; (b) Subpacketization level.

Lemma 3 (Baseline Scheme): Given a positive integer r ($r>1$) and a $(t_1 + 1)$-$(K_1, \binom{K_1}{t_1}, \binom{K_1-1}{t_1-1}, \binom{K_1}{t_1+1})$ MN PDA $\mathbf{P}$, there always exists a (K', r, M', N') D2D assisted multi-access coded caching scheme with the user number of $K' = (r-1)t_1 + K_1$, the cache node memory size of $M' = \frac{N't_1}{(r-1)t_1+K_1}$, the transmission rate of $R' = \frac{\binom{K_1}{t_1+1}(t_1+1)}{\binom{K_1}{t_1}t_1}$ and the subpacketization level of $F' = \binom{K_1}{t_1}t_1[(r-1)t_1 + K_1]$.

Note that user number of the schemes in *Theorem 2* and *Lemma 3* are $K = (r-1)K_0 + t_0$ and $K' = (r-1)t_1 + K_1$, respectively, where K_0 and K_1 denote the number of columns in the corresponding base MN PDAs. For the same user number and node cache size, i.e., $t_0 = t_1$, we have $\frac{K_0}{K_1} = \frac{(r-2)t_0+K_1}{(r-1)K_1}$. This implies that when $r>2$ and $t_0 < K_1$, we have $K_0 < K_1$. Therefore, the ratio of the subpacketization levels between the scheme of *Theorem 2* and that of *Lemma 3* is $\frac{F}{F'} = \frac{\binom{K_0}{t_0}t_0[(r-1)K_0+t_0]}{\binom{K_1}{t_0}t_0[(r-1)t_0+K_1]} = \frac{\binom{K_0}{t_0}}{\binom{K_1}{t_0}} < 1$. Hence, the subpacketization level of the proposed scheme is smaller than that of the scheme of *Lemma 3*.

The following lemma shows that the discrepancy between the achievable rate of the proposed scheme and the lower bound of [15] is within a constant factor determined by r.

Lemma 4: The discrepancy between the achievable rate R of the proposed scheme and the lower bound R^* satisfies $\frac{R}{R^*} < \frac{r(r+1)}{r-1}$.

Proof: Note that under the constraints of $K \leq N$ and uncoded placement, the theoretical lower bound on the D2D transmission rate is $R^* = \frac{\binom{K-1}{t}}{\binom{K-1}{t-1}}$ and the node cache size is $M = \frac{Nt}{K}$, where t is a positive integer that is smaller than K. Let $M^* = rM$ denote the total number of files that each user can access. For the same number of users $K = (r-1)K_0 + t_0$ and M^*, i.e., $t = rt_0$, we have $\frac{R}{R^*} = \frac{r[(K_0-t_0)r-1]}{(r-1)(K_0+t_0)} < \frac{r[(K_0-t_0)r-1+K_0-t_0+1]}{(r-1)(K_0-t_0)} = \frac{r(r+1)}{r-1}$. ■

In order to verify the above characterizations, the subpacketization and the transmission rate performance of the schemes in *Theorem 2*, *Lemma 3* and [15] are compared numerically. Their subpacketization level F, the total number of user accessible files M^* and transmission rate R are shown in Fig. 2. It can be seen that when compared with the baseline scheme, the proposed scheme prevails in both the subpacketization level and the transmission rate. Furthermore, Fig. 2(a) shows that as M^* increases, transmission rate of the proposed scheme converges to the lower bound. This indicates that when the cache size is large, transmission rate of the proposed scheme can reach the theoretical optimum. However, when M^* is small, transmission rate of the proposed scheme remains above the lower bound. This is because the proposed network model imposes more constraints than that of the conventional D2D coded caching. Note that the conventional D2D communication networks allow for arbitrary design of user accessible data. However, in the proposed D2D assisted multi-access networks, this flexibility cannot be accommodated.

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